Vectors, Kinematics, and Delta-V

VIII National Seminar on Traffic Crash Accidents
Brasilia, Brasil
August 2012

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New technology has given us additional tools for the reconstruction of crashes. Information may sometimes be accessed from the vehicle. EDR data may be imaged to provide information about vehicle speed and/or change in velocity at impact. Vehicle speed and/or $\Delta v$ are primary concerns for the crash investigator.
In this presentation, we will look at how applying fundamental physics (Mechanics) along with ACM/EDR data will assist us in reconstructing the crash.

- Newton’s Three Laws of Motion
- Vectors and Kinematics
- Conservation of Linear Momentum
  - Fundamental Assumptions
- Damage Momentum (Using Δv)
Most of us can recite Newton’s Three laws:

- **First**: “A body at rest tends to remain at rest and a body in motion tends to remain in motion unless acted upon by an external, unbalanced force”.

- **Second**: “The acceleration of a body is directly proportional to the unbalanced force acting on the body and is inversely proportional to the mass of the body”.

- **Third**: “For every action (force) there is an equal but opposite reaction (reacting force)”.
We need to realize Newton’s Laws of Motion are actually **vector** equations.

**Newton’s Second:**

\[ F = ma \]

In this equation, \( F \) is the net resultant force acting, and \( a \) is the acceleration of mass \( m \) because of this applied force.

When we multiply a **vector** by a scalar, we obtain a new **vector** with the same direction as the first, but a different magnitude.

In other words, the acceleration will always be in the same direction as the net resultant force.
Newton’s Laws of Motion

We need to realize Newton’s Laws of Motion are actually **vector** equations.

- Newton’s Third:

\[ F_1 = -F_2 \]

In this equation, \( F_1 \) is the acting force and \( F_2 \) is the reacting force.
Kinematics is the study of motion without regard to the cause of the motion.

In essence, it studies the motion of one object with respect to another.

Kinematic definition of $\Delta v$:

$$\Delta v = v_{\text{final}} - v_{\text{initial}}$$

$$\Delta v = v_3 - v_1$$
When we reconstruct traffic crashes, we use two different coordinate systems:

- Inertial coordinates are attached to the earth and never accelerate.

- Non-inertial coordinates are attached to the objects, and thus accelerate with the objects.
  - For example, a Cartesian coordinate system attached to the center of mass of a vehicle has +x forward, +y to the right, and +z down.
  - This moves and rotates with the vehicle.
Kinematics and $\Delta v$

Non-inertial coordinates attached to a vehicle
Consider the following problem:

- A vehicle skids for 30 m with a drag factor of 0.70. It then impacts a tree at a speed of 50 km/h.
  - What is its initial speed?
  - What is the $\Delta v$ across the surface?
  - What is the $\Delta v$ at the tree if the vehicle hits and stops?
The first question is answered simply by the combined speed equation:

\[ S = \sqrt{254df + S_i^2} \]

\[ S = \sqrt{254(30)(0.70) + 50^2} \]

\[ S = \sqrt{7834} \]

\[ S = 88.50 \text{ km/h} \]
The second question is answered with kinematics: Let the initial direction be positive.

\[
\Delta v = v_{\text{final}} - v_{\text{initial}}
\]

\[
\Delta v = 50 - 88.50
\]

\[
\Delta v = -38.5 \text{ km/h}
\]

Note the sign on \(\Delta v\) is negative, indicating the direction of this vector.
The third question is also answered with kinematics: Let the initial direction be positive.

\[ \Delta v = v_{\text{final}} - v_{\text{initial}} \]

\[ \Delta v = 0 - 50 \]

\[ \Delta v = -50 \text{ km/h} \]

Note the sign on \( \Delta v \) is negative, indicating the direction of this vector.
\( \Delta v \) or KEES (EBS)?

**Definitions:**

- \( \Delta v \) is the vector change in velocity. It is computed via the kinematics of the motion.
- KEES (Kinetic Energy Equivalent Speed) and EBS (Equivalent Barrier Speed) represent the amount of kinetic energy dissipated by an event.
  - For example, KEES would represent the KE dissipated while skidding across a surface.
  - EBS would represent the KE dissipated by the crushing of a vehicle striking a barrier.
Let’s examine our skid to tree problem

The KEES for the skid is:

\[ S = \sqrt{254 \cdot df} \]

\[ S = \sqrt{254(30)(0.70)} \]

\[ S = 73.03 \text{ km/h} \]

Notice the magnitude of the KEES is significantly different than the \( \Delta v \) of -24.77 mph across the first surface. The KEES is a scalar quantity, as it represents the scalar KE dissipated across that surface.
Will the $\Delta v$ and KEES (EBS) ever be the same magnitude?

- YES…If the final velocity is zero.
- For our example, both the EBS and $\Delta v$ at the tree impact have the same magnitude of 50 km/h.
Where do We Get $\Delta v$?

- **EDR**
  - Imaged data from air bag control module
  - User specified direction in many cases
  - Must reconcile with actual $\Delta v$ using collision geometry.

- **CRASH III (Damage Momentum) Analysis**
  - Calculates magnitude of the $\Delta v$ vector based upon vehicle damage.
    - Analyst must specify the direction of the vector.
    - Most $\Delta v$ data points in the mid-50’s speed range
Traditional COLM

- Calculates both the magnitude and direction of the $\Delta v$ vector
- These methods may be used as a check on each other.
A Tool for Collision Analysis

CONSERVATION OF LINEAR MOMENTUM
Newton's Second Law says...

- Force = (mass) x (acceleration)

\[ F_1 = m_1 a_1 \]

\[ F_2 = m_2 a_2 \]
Newton’s Third Law says...

- Forces are equal and opposite

\[ F_1 = m_1a_1 \]
\[ F_2 = m_2a_2 \]
\[ F_1 = -F_2 \]
\[ m_1a_1 = -m_2a_2 \]
Definition of Acceleration

**acceleration =**

\[ a_i = \frac{\Delta v_i}{\Delta t} \]

\[ F_1 = -F_2 \]

\[ m_1 a_1 = -m_2 a_2 \]

\[ m_1 \left( \frac{\Delta v_1}{\Delta t} \right) = -m_2 \left( \frac{\Delta v_2}{\Delta t} \right) \]
Forces & times acting on each vehicle are the same, so $\Delta P$ equal and opposite.

$$F = ma = m\left(\frac{\Delta V}{\Delta t}\right)$$

$$F(\Delta t) = m(\Delta V) \quad \text{Impulse} = \text{change in momentum}$$

So: $\Delta M = m(\Delta V) = F(\Delta t)$
Conservation Of Linear Momentum: Linear Momentum IN = Linear Momentum OUT

\[ m_1 \Delta v_1 = -m_2 \Delta v_2 \]

\[ m_1 (v_3 - v_1) = -m_2 (v_4 - v_2) \]

\[ m_1 v_3 - m_1 v_1 = -m_2 v_4 + m_2 v_2 \]

\[ m_1 v_1 + m_2 v_2 = m_1 v_3 + m_2 v_4 \]
In the classic COLM equation we just derived, the ONLY forces acting are the collision forces!

- Ground frictional forces as well as aerodynamic forces are ignored.
  - These are usually small compared to the collision forces if the vehicles are reasonably similar in size.
  - Caution must be used in low speed (low force) collisions or in impacts where external impulsive forces may be significant.
Because there is no potential energy change in a collision, an elastic collision is defined as one in which kinetic energy is conserved.
An inelastic collision is simply one in which Kinetic Energy is NOT conserved.
The work done to crush the vehicles is irreversible work.

Essentially, this means the work, hence energy, used to crush the vehicles is transformed into other forms of energy, such as heat.
We consider traffic crashes at normal street and highway speeds to be inelastic.

Our experience with controlled testing over the years tells us this is a reasonable assumption.

However, some low-speed collisions will have some “bounce” to them.

We describe this as the coefficient of restitution.
Consider for a moment a collinear, central collision between two bodies.

Newton defined the coefficient of restitution as follows:

$$\varepsilon = \frac{v_4 - v_3}{v_1 - v_2}$$
Where:

\[ \varepsilon = \text{coefficient of restitution} \]

\[ v_4 = \text{post-impact velocity of body 2} \]

\[ v_3 = \text{post-impact velocity of body 1} \]

\[ v_1 = \text{impact velocity of body 1} \]

\[ v_2 = \text{impact velocity of body 2} \]

This is also known as the *kinematic* definition of restitution
Exercise: Calculate the coefficient of restitution of a vehicle hitting a barrier at 50 km/h and recoiling at 5 km/h. Initial direction is positive.

\[ \varepsilon = \frac{v_4 - v_3}{v_1 - v_2} \]

\[ = \frac{0 - (-5)}{50 - 0} \]

\[ = 0.10 \]
Follow-up: What is the $\Delta v$ of this vehicle?

Using kinematics:

$$\Delta v = v_{\text{final}} - v_{\text{initial}}$$

$$\Delta v = -5 - 50$$

$$\Delta v = -55 \text{ km/h}$$
We may apply the coefficient of restitution with the following equation:

\[ v_{\text{close}} = \frac{\Delta v_1 (w_1 + w_2)}{w_2 (\varepsilon + 1)} \]
Where:

\[ v_{\text{close}} = \text{closing velocity} \ (v_1 - v_2) \]
\[ \Delta v_1 = \text{Velocity change of body 1} \]
\[ m_1 = \text{mass of body 1} \]
\[ m_2 = \text{mass of body 2} \]
\[ \varepsilon = \text{coefficient of restitution} \]

This equation may be useful when \( \Delta v_1 \) is known, perhaps from an event data recorder or a damage momentum analysis.
Some typical values for Coefficient of Restitution:

- $\Delta V$ above 15-20 mph: 0.0 to 0.15
- $\Delta V$ below 15 mph: 0.15 to 0.45

In low speed collisions, we will have to account for a coefficient of restitution. If vehicles become entangled, then $\varepsilon = 0$
Planar Collisions

- Planar collisions are so named because the collision takes place on a *plane*.
- The approach velocity vectors of the vehicles may be parallel to each other (collinear).
- If the approach velocity vectors are not parallel and make some angle with respect to each other, then we analyze in two dimensions.
  - Sometimes referred to as 360° momentum
- The collision can be central or non-central.
Traditional COLM Analysis

- We know:
  - Post Impact Directions
  - Post Impact Speeds
  - Approach Directions
  - Vehicle Weights

- We define a system:
  - External, impulsive forces are minimal
  - Choose the approach of one vehicle on the x-axis
    - Done for our convenience
    - Not dictated by the Principle of COLM
COLM is a vector analysis

Need to know magnitudes and directions of at least some of the vectors

Some crashes may not have all “traditional” evidence present.

If we have a basic understanding of planar vector analysis, we may often use the evidence we do have to find a solution.

Let’s look at an example...
Example:

Unit #1 is traveling eastbound on Main St., Unit #2 is traveling northbound on Ash St. Both units collide in the intersection at a right angle with Unit #1 departing the collision at an angle of $40^\circ$ and Unit #2 departing the collision at an angle of $25^\circ$. Unit #1’s departure speed was 50 km/h and Unit #2’s departure speed was 32 km/h.

Unit #1 weighs 1364 kg, and Unit #2 weighs 909 kg.

Determine the impact speeds $v_1$ and $v_2$.

Determine $\Delta v_1$ and $\Delta v_2$.

Determine the P.D.O.F. angles, $\alpha_1$ and $\alpha_2$. 
The Workhorse Equations

\[ v_2 = \frac{w_1 v_3 \sin \theta}{w_2 \sin \psi} + \frac{v_4 \sin \phi}{\sin \psi} \]

\[ v_1 = v_3 \cos \theta + \frac{w_2 v_4 \cos \phi}{w_1} - \frac{w_2 v_2 \cos \psi}{w_1} \]

(Veh.1 pre-crash direction = 0 degrees)
The Workhorse Equations: Solve for $v_2$ first

\[ v_2 = \frac{w_1 v_3 \sin \theta}{w_2 \sin \psi} + \frac{v_4 \sin \phi}{\sin \psi} \]

Do the Math:

\[ v_2 = 61.70 \text{ km/h} \]
The Workhorse Equations: Solve for $v_1$ next

\[
v_1 = v_3 \cos \theta + \frac{w_2 v_4 \cos \phi}{w_1'} - \frac{w_2 v_2 \cos \psi}{w_1'}
\]

Do the Math:

\[
v_1 = 57.62 \text{ km/h}
\]
Consider this acute triangle: What if we know one angle and the two adjacent sides? The *Law of Cosines* will compute the other side. For example, what if we know angle C and sides a & b and want to find side C?

\[ c = \sqrt{a^2 + b^2 - 2ab \cos C} \]
Consider this acute triangle: What if we know two angles and one side? The Law of Sines will compute the remaining sides. For example, what if we know angles C and A and side a and want to find side c?

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

Thus: \( c = \frac{a \sin C}{\sin A} \)
Calculate $\Delta V$ (Law of Cosines)

\[ \Delta v_1 = \sqrt{v_1^2 + v_3^2 - 2v_1v_3 \cos \theta} \]
\[ \Delta v_2 = \sqrt{v_2^2 + v_4^2 - 2v_2v_4 \cos(\psi - \varphi)} \]

- $v_1 = 57.62$ km/h
- $v_3 = 50$ km/h
- $\Theta = 40^\circ$
- Do the Math: $\Delta v_1 = 37.50$ km/h

- $v_2 = 61.70$ km/h
- $v_4 = 32$ km/h
- $(\Psi - \Phi) = (90 - 25)^\circ = 65^\circ$
- Do the Math: $\Delta v_2 = 56.23$ km/h
Show each vehicle’s $\Delta v$.

Unit #1: $\Delta v_1 = 37.50$ km/h in the $\Delta P_1$ direction
Unit #2: $\Delta v_2 = 56.23$ km/h in the $\Delta P_2$ direction
PDOF Angles (Law of Sines)

\[ \alpha_1 = \sin^{-1} \left[ \frac{v_3 \sin \theta}{\Delta v_1} \right] \]

\[ \alpha_2 = \sin^{-1} \left[ \frac{v_4 \sin(\psi - \varphi)}{\Delta v_2} \right] \]
Calculate PDOF Angles

\[ \alpha_1 = \sin^{-1} \left[ \frac{v_3 \sin \theta}{\Delta v_1} \right] \]

\[ \alpha_2 = \sin^{-1} \left[ \frac{v_4 \sin (\psi - \varphi)}{\Delta v_2} \right] \]

- \( v_3 = 50 \text{ km/h} \)
- \( \Delta v_1 = 37.50 \text{ km/h} \)
- \( \theta = 40^\circ \)
- Do the Math:
  - \( \alpha_1 = 58.87^\circ \)

- \( v_4 = 32 \text{ km/h} \)
- \( \Delta v_2 = 56.23 \text{ km/h} \)
- \( (\psi - \varphi) = 65^\circ \)
- Do the Math:
  - \( \alpha_2 = 31.05^\circ \)
PDOF Convention
\[ \alpha_2 = -31^\circ \]

Show PDOF angles.

\[ \alpha_1 = 59^\circ \]
We just solved a crash using COLM in the traditional manner:
- Known Approach and Departure angles
- Known weights
- Known Post – Impact Velocities

What if we don’t have that evidence?
We will examine the same crash with different evidence.
Consider this evidence:

- Known approach and departure angles
- Longitudinal $\Delta v_1$ (19.31 km/h)
- $\text{PDOF}_1 \approx 59^\circ$ (User Specified)

We do not have a post impact speed for either vehicle

Consider the following diagram, which is for Vehicle 1:
The PDOF represents the direction of the $\Delta v_1$ vector. The EDR can only compute the forward velocity change. In this case, there is both a local $x$ and local $y$ velocity change.

If we examine the geometry, we see the following relationship:

$$\Delta v_1 = \frac{\Delta v_{1x}}{\cos \alpha_1}$$

$$\Delta v_1 = \frac{19.31}{\cos 59} = 37.50 \text{ km/h}$$
Calculate angle C:

\[ C = 180 - (\alpha_1 + \theta) \]

\[ C = 81^\circ \]

\[ \Delta v_1 = 37.50 \text{ km/h} \]

Use the Law of Sines to compute \( v_1 \):

\[ v_1 = \frac{\Delta v_1 \sin(C)}{\sin \theta} \]

\[ v_1 = \frac{37.50 (\sin 81)}{\sin 40} = 57.62 \text{ km/h} \]
Vehicle 2 Collision Diagram

Calculate the magnitude of $\Delta v_2$ using Newton’s 3rd Law:

\[
\Delta v_2 = \frac{m_1}{m_2} \Delta v_1 = \frac{1364}{909} \times 37.50 = 56.27 \text{ km/h}
\]

Calculate the $v_2 - v_4$ angle, $A$:

\[
A = \psi - \phi = 90 - 25 = 65^\circ
\]

Calculate $\alpha_2$:

\[
\alpha_2 = 180 - (\alpha_1 + \psi) = 180 - (59 + 90) = 31^\circ
\]
\[ \alpha_2 = 31^\circ \]

**Vehicle 2 Collision Diagram**

Calculate angle B:

\[ B = 180 - (\alpha_2 + A) \]

\[ B = 180 - (31 + 65) = 84^\circ \]

Calculate \( v_2 \) using the Law of Sines:

\[ v_2 = \frac{\Delta v_2 \sin(180 - (\alpha_2 + A))}{\sin(A)} \]

\[ v_2 = \frac{56.27 \sin(84^\circ)}{\sin 65^\circ} = 61.75 \text{ km/h} \]
Consider this evidence:

- Known approach and departure angles
- Longitudinal $\Delta v_{1x}$ (19.31 km/h)
- Lateral $\Delta v_{1y}$ (32.14 km/h)
  - $\alpha_1 = 59^\circ$ from EDR Data
  - Heading and bearing are collinear

We do not have a post impact speed for either vehicle

Consider the following diagram, which is for Vehicle 1:
The PDOF represents the direction of the $\Delta v_1$ vector. The EDR can now compute the forward and lateral velocity changes.

If we examine the geometry, we see the following relationship:

$$\Delta v_1 = \sqrt{19.31^2 + 32.14^2}$$

$$\Delta v_1 = 37.49 \text{ km/h}$$
Reconciling the EDR PDOF with the Collision Force Angle for the Crash III magnification factor: In this case, figure a side impact.

The Collision Force Angle is measured with respect to a line normal to the damage face.

\[ \alpha_1 = 59^0 \]

\[ \alpha = 31^\circ \]

\[ Mag = 1 + \tan^2 \alpha \]

\[ Mag = 1.36 \]
Vehicle 1 Collision Diagram

Calculate angle C:

\[ C = 180 - (\alpha_1 + \theta) \]

\[ C = 81^\circ \]

\[ \Delta v_1 = 37.49 \text{ km/h} \]

\[ \alpha_1 = 59^\circ \]

\[ v_1 = \frac{\Delta v_1 \sin(C)}{\sin \theta} \]

\[ v_1 = \frac{37.49 \sin(81)}{\sin 40} = 57.61 \text{ km/h} \]

Use the Law of Sines to compute \( v_1 \):
We will look at Vehicle 1 with some slightly different evidence.

- We now have a reasonable post-impact speed for it, as well as its post-impact direction.

- All other evidence as the second scenario…
Use the Law of Cosines to compute $v_1$:

$$v_1 = \sqrt{v_3^2 + \Delta v_1^2 - 2v_3\Delta v_1 \cos(180 - (\theta + \alpha_1))}$$

$$v_1 = \sqrt{2500 + 1405.5 - 2(50)(37.49)\cos(81)}$$

$$v_1 = 57.61 \text{ km/h}$$
We will examine the case where Vehicle 1 comes into the crash with a slip angle of 10°.

- We have a cumulative longitudinal $\Delta v_{lx}$ of 13.44 km/h

- We will examine the diagram on the next slide…
The PDOF represents the direction of the $\Delta v_1$ vector. The EDR can only compute the forward velocity change. In this case, there is both a local $x$ and local $y$ velocity change. In this case, we have a slip angle $\beta$.

If we examine the geometry, we see the following relationship:

$$\Delta v_1 = \frac{\Delta v_{1x}}{\cos(\alpha_1 + \beta)}$$

$$\Delta v_1 = \frac{13.44}{\cos 69} = 37.50 \text{ km/h}$$
Compute Collision Force Angle with respect to the EDR and then the PDOF angle with respect to the \( v_1 \) vector.

\[
(\alpha_1 + \beta) = \tan^{-1} \frac{35.00}{13.44}
\]

\[
(\alpha_1 + \beta) = 69^\circ
\]

\[
\alpha_1 = 69^\circ - 10^\circ = 59^\circ
\]
The PDOF represents the direction of the $\Delta v_1$ vector with respect to the $v_1$ vector. The EDR can compute the forward and lateral velocity change with respect to its orientation aligned with the vehicle.

$\Delta v_{1y} = 35.00 \text{ km/h}$

$\Delta v_{1x} = 13.44 \text{ km/h}$

$\Delta v_1 = \sqrt{13.44^2 + 35.00^2}$

$\Delta v_1 = 37.49 \text{ km/h}$

$\beta = 10^\circ$

$\alpha_1 = 59^\circ$
Vehicle 1 Collision Diagram with slip angle

ACM/EDR with lateral and longitudinal sensors

Finally, compute Collision Force Angle, \( \alpha \), with respect to the side damage face in order to calculate the CRASH III magnification factor.

\[ \alpha = 90° - 69° \]

\[ \alpha = 21° \]

\[ Mag = 1 + \tan^2 \alpha \]

\[ Mag = 1.15 \]
Example: Head-on with $\Delta v$

In this crash, a 2273 kg vehicle heading east hits a 1818 kg vehicle headed west. The collision is central, and both units move off as one for a distance of 10.66 m with an overall drag factor of 0.49. The positive direction is to the right.

A CRASH III (Damage Momentum) analysis shows a $\Delta v_1$ magnitude of 47.53 km/h. This vector will be negative, since the positive motion is slowed.

Calculate $\Delta v_2$, $v_{\text{common}}$, $v_1$ and $v_2$, keeping directions in mind.
Example: Head-on with $\Delta v$

Calculate $\Delta v_2$ using Newton’s Third Law:

$$\Delta v_2 = -\frac{m_1}{m_2} \Delta v_1 = -\frac{2273}{1818}(-47.53) = +59.42 \text{ km/h}$$
Calculate $v_{\text{common}}$ using the slide to stop equation:

$$S = \sqrt{254df}$$

$$S = \sqrt{254(10.66)(0.49)}$$

$S = 36.42 \text{ km/h}$

Inspection tells us this is the + direction
Calculate $v_1$ using the kinematic definition of $\Delta v$:

$$\Delta v_1 = v_{\text{common}} - v_1$$

$$v_1 = v_{\text{common}} - \Delta v_1$$

$$v_1 = 36.42 - (-47.53)$$

$$v_1 = 83.95 \text{ km/h}$$
Calculate $v_2$ using the kinematic definition of $\Delta v$:

$\Delta v_2 = v_{\text{common}} - v_2$

$v_2 = v_{\text{common}} - \Delta v_2$

$v_2 = 36.42 - (+59.42)$

$v_2 = -23 \text{ km/h}$

The negative sign on the speed simply means the second vehicle was westbound at impact, which is the negative direction in the coordinate system chosen.
We may check the results with an appropriate COLM equation:

\[ m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_{\text{common}} \]

\[ 2273(83.95) + 1818(-23) = 4091(36.42) \]

\[ 190,818 - 41,814 \approx 148,994 \]

\[ 149,004 \approx 148,994 \]
As an exercise, calculate the closing velocity using the closing velocity equation:

\[ v_{\text{close}} = \frac{\Delta v_1 (m_1 + m_2)}{m_2 (\varepsilon + 1)} \]

\[ v_{\text{close}} = \frac{47.53(4091)}{1818(0+1)} \]

\[ v_{\text{close}} = 106.96 \text{ km/h} \]

Same as the closing velocity from a vector addition.
Consider the following scenario:

- A school bus pulls out of a side road from a stop and onto the first lane of a four lane highway.
- The bus is hit in the side, near center, by a Honda CRX, which penetrates back to the C pillar.
- The bus driver continues across the road in a left hand turn and parks on the shoulder, still wearing the CRX.
- The CRX leaves 41.45 m of locked wheel skids to impact. The skids show no sideslip. The drag factor is 0.72.
- The hood of the CRX shows scratches bearing yellow paint, which make an angle of 14° to the right with respect to the hood.
- Testing of the bus showed a likely impact speed for it of 16 km/h at an angle of 100° with respect to the CRX skid.
Make the following calculations:
Speed of the CRX at impact

\[ v_{CRX} = \frac{v_b \sin C}{\sin(B)} \]

\[ v_2 = \frac{16(\sin 66)}{\sin 14} = 60.42 \text{ km/h} \]

Speed of the CRX at the beginning of the skid

\[ S = \sqrt{254 df + S_i^2} \]

\[ S = \sqrt{254 (41 \cdot 45)(0.72) + 60.42^2} \]

\[ S = 105.98 \text{ km/h} \]
Newton’s Laws of Motion
- Realize Newton’s Laws are vector equations.

Kinematics studies the motion of one object with respect to another.
- Coordinate systems may be fixed (inertial) or attached to an object or vehicle (non-inertial).
- Kinematics provide a specific, vector definition of $\Delta v$.

Differences between $\Delta v$ and EBS/KEES
- What to ask…is the final velocity zero?
- $\Delta v$ is a vector and follows the rules of vector addition
- EBS/KEES are speeds based on KE dissipation
  - Combined, not added
Origin of $\Delta v$ data:

- EDR image
  - Must be reconciled using collision geometry
  - User specified direction in many cases

CRASH III:

- Uses vehicle damage to compute magnitude of $\Delta v$
- Analyst specifies direction
- Most $\Delta v$ test data points in the mid-50s

Traditional COLM

- Calculates both magnitude and direction of $\Delta v$ based upon collision geometry and post-impact information.

Various methods can check one another
Discussion

- **Traditional COLM**
  - Derived from Newton’s 2nd and 3rd Laws
    - External, impulsive forces considered negligible
    - Impact speeds calculated from post-impact data
    - May be used to calculate $\Delta v$ and PDOF directly

- **Working with $\Delta v$ vectors**
  - Post impact speeds not needed in two dimensional collisions if directions and $\Delta v$ are known.
  - Impact speed analysis by Law of Sines
  - Second vehicle $\Delta v$ derived from first vehicle
  - Impact speed from Law of Cosines if $\Delta v$ and post impact velocity known
Head-On Collision Example

- Post-impact speed known
- $\Delta v_1$ calculated from CRASH III
  - User specified direction
- $\Delta v_2$ calculated from Newton’s Third Law
- Impact speed calculated kinematically

Kinematic Impact Analysis using velocity vectors

- Relative velocity of one vehicle to another
Lots of Tools for the Toolbox!

All based upon known principles of mechanics
References